*International Journal of Theoretical Physics, Vol. 28, No. 9, 1989*

# Gravitons from Anomalous Decay

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*Received June 20, 1989*

The decay rate of the neutral pion into two gravitons is calculated from the gravitational anomaly in the axial current. Although this decay rate is negligible relative to the decay rate of the neutral pion into two photons, the rate of decay into gravitons is proportional to the seventh power of the mass of the decaying particle, and to the square of the gravitational constant. The possibility that a particle of very large mass, associated with an axial current anomaly, was present in the early universe is considered. Such a particle would decay at a significant rate into gravitons. As these gravitons would not be thermalized, they would result in a (potentially observable) nonthermal spectrum of gravitational waves present today. The peak frequency of this gravitational wave spectrum would be indicative of the mass of the decaying particle. Alternatively, if the gravitational constant were large at early times, then the gravitational decay of the pion would be significant in the early universe, giving rise to a nonthermal gravitational wave spectrum.

## 1. INTRODUCTION

The decay of the neutral pion into two photons is the result of an anomaly (Adler, 1969; Bell and Jackiw, 1969) in the divergence of the quark-antiquark axial current $J_A^\mu$. The decay rate for $\pi^0 \to 2\gamma$ given by the anomaly is

$$\Gamma_{em} = \frac{\alpha^2 m^3}{64\pi^3 f_\pi^2} \tag{1}$$

where $\alpha$ is the fine structure constant, $m$ is the mass of the $\pi^0$, and $f_\pi$ is the pion decay constant (I use units with $\hbar = c = 1$). In the case when the gravitational field is described by curvature of spacetime characterized by the Riemann tensor $R^\alpha_{\beta\gamma\delta}$, the covariant divergence of the quark–antiquark

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0020-7748/89-0900-1163$06.00/0 

axial current has the form

$$\nabla_\mu J_A^\mu(x) = f_\pi m^2 \pi(x) - \frac{\alpha}{4\pi}\, \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta}(x) F_{\gamma\delta}(x) + \frac{1}{384\pi^2}\, \varepsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta}{}^{\lambda\sigma}(x) R_{\gamma\delta\lambda\sigma}(x) \tag{2}$$

where $\nabla$ is the curved spacetime covariant derivative. The term involving the Maxwell field $F_{\alpha\beta}$ gives rise to the decay rate of equation (1). This electromagnetic term arises purely as a result of renormalization (Adler, 1969; Bell and Jackiw, 1969) and is therefore referred to as an anomaly. The gravitational term involving the Reimann tensor arises in the same way (Kimura, 1969; Delbourgo and Salam, 1972; Eguchi and Freund, 1976). It should cause the decay of the $\pi^0$ into two gravitons. However, perhaps because the gravitational decay rate is expected to be so small in comparison with the electromagnetic decay rate, no one to my knowledge has explicitly calculated it. I have carried out the calculation of the gravitational decay rate resulting from the gravitational anomaly for two reasons. First, the decay is a true quantum gravitational effect. If there were some way in which one could observe the properties of this anomalous mode in the decay of a particle, then one could infer the existence of quanta of the gravitational field. The explicit expression for the decay rate into gravitons will suggest hypothetical scenarios in which the anomalous decay into gravitons would compete with the electromagnetic decay mode, with possibly observable consequences. Second, the one-loop result for the electromagnetic anomaly is not altered or renormalized by higher-order contributions. Although the effect of higher-order contributions in a quantum theory of gravity is not clear because of problems with renormalizability, one can reasonably expect, as in the electromagnetic case, that the gravitational anomaly and associated decay rate will not be altered by higher-order contributions.

## 2. DECAY RATE OF THE NEUTRAL PION INTO GRAVITONS

In order to calculate the decay rate or inverse lifetime for a $\pi^0$ at rest to decay into a pair of gravitons, let us consider a flat background metric $\eta_{\mu\nu}$ with quantized metric perturbations $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3}$$

I will omit the details of the calculation, concentrating mainly on the results. The decay rate is given by an expression of the form (Itzykson and Zuber, 1980, p. 553)

$$\Gamma=\frac{1}{2m}\sum_{\varepsilon_1,\varepsilon_2}\int\frac{d^3k_1\,d^3k_2}{4(2\pi)^6\omega_1\omega_2}|\mathcal{T}|^2(2\pi)^2\delta^{(4)}(q-k_1-k_2) \tag{4}$$

where

$$\mathcal{T}=\lim_{q^2\to m^2}(m^2-q^2)<h(k_1,\varepsilon_1),h(k_2,\varepsilon_2)|\pi(x=0)|0\rangle \tag{5}$$

Here, the initial state contains a pion at rest, and the final state contains two gravitons (excitations of the $h_{\mu\nu}$ field). The four-momentum of the pion is denoted by $q$ and the four-momenta and polarizations of the gravitons by $(k_1, \varepsilon_1)$ and $(k_2, \varepsilon_2)$, respectively. Solving equations (2) for $\pi(x)$ and substituting into equation (5), one finds that the nonvanishing contribution to this matrix element comes from the Riemann tensor term. As in the electromagnetic case, I assume that the matrix element has a simple pole at $q^2=m^2$, and that the value of $\mathcal{T}$ at $q^2=m^2$ is well approximated by its value at $q^2=0$. One then finds, after a lengthy calculation, that

$$\mathcal{T}=-\frac{Gm^4}{12\pi f_\pi} \tag{6}$$

where $G$ denotes the Newtonian gravitational constant. Finally, equation (4) yields the decay rate of a $\pi^0$ at rest into two gravitons as

$$\Gamma=\frac{G^2m^7}{2304\pi^4f_\pi^2} \tag{7}$$

I next briefly discuss two scenarios in which such decay into gravitons may be of significance.

## 3. MASSIVE PARTICLE AND NONTHERMAL GRAVITON BACKGROUND

Note that the gravitational decay rate increases much more rapidly with mass than does the electromagnetic decay rate of equation (1). As expected, the ratio of the gravitational to electromagnetic decay rate is very small:

$$\frac{\Gamma}{\Gamma_{em}}=\frac{G^2m^4}{36\pi\alpha^2} \tag{8}$$

However, the strong dependence of the gravitational decay rate on mass suggests the following speculation. Suppose that there is a massive neutral particle appearing as an interpolating field in the divergence of an axial current, which contains gravitational and electromagnetic anomalies, as in equation (2). In that case, the electromagnetic and gravitational decay rates would be given by equations (1) and (7), respectively, but with $m$ replaced by the much larger mass of the decaying particle, and with $f_\pi$ replaced by an appropriate constant. The constant $f_\pi$ does not appear in the ratio of equation (8).

In terms of the Planck mass, $m_{\rm Pl} = (\hbar c/G)^{1/2}$, this ratio is

$$\Gamma/\Gamma_{em} = (36\pi\alpha^2)^{-1}(m/m_{\rm Pl})^4 \tag{9}$$

which is unit when $m = 0.28 m_{\rm Pl}$. If there exist such particles having a mass approaching $0.28 m_{\rm Pl}$, they could be created in the earliest stage of the big bang. They would go out of equilibrium at an early time, and (assuming that other decay modes do not totally dominate the graviton and photon decay modes) would decay at a significant rate into gravitons. These gravitons would not be thermalized, and their redshifted frequency would be related to the mass of the particle which had decayed. On the other hand, the photons resulting from the associated electromagnetic decay would be thermalized, increasing the temperature of the ambient radiation background. With rapid progress being made in gravitational wave detection, one may expect that eventually gravitational waves from such decays in the early universe, if they exist, will be observable.

## 4. VARIABILITY OF $G$ AND NONTHERMAL GRAVITON BACKGROUND

In theories in which the effective gravitational constant $G$ varies with time in units in which $e$, $h$, and particle masses are constant (Dirac, 1979),[2] the ratio in equation (8) would, on the basis of the large-numbers hypothesis, approach unity at early times, and the decay rate of equation (7) would become significant, even for the pion mass. Then we need not postulate the existence of unknown particle species in order to obtain the nonthermal graviton background. Ordinary pions produced at early times would decay via the gravitational anomaly at a significant rate into graviton pairs. The graviton energy at the time of decay could be peaked at close to one-half the rest energy of the decaying pions. Thus, by observing the spectrum of nonthermal gravitons in the cosmic gravitational wave background, one may be able to extract information about the variability of the gravitational constant.

[2]The variable-rest-mass theories considered and referenced in Bekenstein (1977) may also be regarded as variable-$G$ theories in appropriate units, and under certain conditions $G$ can become large in the early universe in their theories.

## 5. CONCLUSIONS

I have calculated the decay rate of the neutral pion into gravitons, which results from the gravitational anomaly in the axial current. From the dependence of the decay rate on mass, I find that if there were an analogous particle having mass approaching a fraction of the Planck mass, then its decay into gravitons via the anomaly would result in a nonthermal cosmic gravitational wave background at a frequency characteristic of the rest energy of the decaying particle. Alternatively, if the gravitational constant measured in atomic units were large in the early universe, then a similar nonthermal cosmic gravitational wave background would be present from the early decay of pions into gravitons. In that case, the peak frequencies in the spectrum would be related to the pion rest mass.

## ACKNOWLEDGMENT

I thank the National Science Foundation for support under grant PHY-8603173.

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